

ESTIMATION OF THE EFFICIENCY OF THE LITHUANIAN FAMILY FARMS VIA THE ORDER-M FRONTIERS

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The paper aimed at analysing the patterns of efficiency of the Lithuanian family farms with respect to the uncertain data. The latter aim was achieved by the virtue of the probabilistic production functions. The sensitivity of the efficiency scores estimated for the Lithuanian family farms was analysed by manipulating the numbers of randomly drawn benchmark observations estimations and thus constructing respective order-*m* frontiers. The livestock farms appeared to be most efficient, or even super-efficient, independently of the model orientation or the order of the frontier. The crop farms exhibited the lowest mean efficiency as well as the widest distribution of the efficiency scores. The mixed farming was more related to the livestock farming in case of the input-oriented framework and to the crop farming in the output-oriented one in terms of the mean efficiency scores. Thus, the mixed farms were located inside the production frontier (surface) in a rather compact way.

Keywords: efficiency, family farms, order-*m* frontier, activity analysis.

JEL codes: C140, C440, D240, Q120.

Introduction

The measurement of the efficiency of agricultural sector is an important issue for scientific studies due to various circumstances. First, the agricultural sector is related to voluminous public support. Second, a significant share of the rural population is employed in the agricultural sector. This is particularly the case in Central and Eastern European countries, where agricultural sectors are even more important for the local economies. The measures of efficiency enable to describe the state of agricultural sector as well as identify the means for improvement (Mendes, 2013).

The analyses of efficiency and productivity usually rest on the estimation of the production frontier. The production frontier can be estimated via either the parametric or non-parametric methods or combinations thereof. The non-parametric techniques are appealing ones due to the fact that they do not need the explicit assumptions on the functional form of the underlying production function and still enable to impose certain axioms in regards to the latter function (Afriat, 1972).

The deterministic non-parametric methods, though, feature some caveats. Given the data generating process (DGP) of the observed production set is unknown, the underlying production set also remains unknown. Therefore, the efficiency scores based on the observed data, i. e. a single realization of the underlying DGP, might be

biased due to outliers. As a remedy to the latter shortcoming, the statistical inference could be employed to construct the random production frontiers.

The partial frontiers (also referred to as the robust frontiers) were introduced by C. Cazals et al. (2002). The idea was to benchmark an observation not against all the observations dominating it but rather against a randomly drawn sample of these. This type of frontier was named the order- m frontier. The latter methodology has been extended by introducing the conditional measures enabling to analyse the impact of the environmental variables on the efficiency scores (Daraio, 2005, 2007a, 2007b). D. C. Wheelock and P. W. Wilson (2003) introduced the Malmquist productivity index based on the partial frontiers. L. Simar and A. Vanhems (2013) presented the directional distance functions in the environment of the partial frontiers. The order- m frontiers have been employed in the sectors of healthcare (Pilyavsky, 2008) and finance (Abdelsalam, 2013) among others.

In spite of the importance of the efficiency analysis and the shortcomings of the conventional efficiency measures, efficiency of the Lithuanian agricultural sector—like that of the other ones—has not been analysed by the means of the partial frontiers. Indeed, the Lithuanian agricultural sector has been analysed by the means of the bootstrapped Data Envelopment Analysis (Baležentis, 2012). However, the latter method offers rather poor means for the analysis of sensitivity. Therefore, there is a need for further analyses of performance of the Lithuanian family farms and agricultural sector in general. The simulation-based methodology is of particular importance in the latter context.

This paper, therefore, aims at analysing the patterns of efficiency of the Lithuanian family farms with respect to the uncertain data. The latter aim was achieved by the virtue of the order- m frontiers. The following tasks were set: 1) to describe the methodology of the order- m frontiers; 2) to employ the order- m frontiers for estimation of the efficiency scores for the Lithuanian family farms; 3) to perform sensitivity analysis of the obtained results.

The paper is structured in the following manner: Section 1 presents the computations associated with the order- m frontiers. Section 2 describes the data used. Finally, the results are presented in Section 3.

1. Preliminaries for the estimation of the order- m frontiers

The activity analysis (Koopmans, 1951; Debreu, 1951) defines the production technology by treating the sets of inputs, $x \in \mathfrak{R}_+^p$, and outputs, $y \in \mathfrak{R}_+^q$, across the decision making units (DMUs). The technology set, T , consists of all feasible production plans:

$$T = \{(x, y) \in \mathfrak{R}_+^{p+q} \mid x \text{ can produce } y\}. \quad (1)$$

Furthermore, the free disposability of inputs and outputs is assumed (Shepard, 1970), i. e. $(x, y) \in T \Rightarrow (x', y') \in T$ for $x \leq x', y' \leq y$. Note that inequalities between vectors are to be read element-wise throughout the paper.

The input- and output-oriented Farrell measures of efficiency can be defined, respectively, as (Farrell, 1957):

$$\theta(x, y) = \inf \{ \theta \mid (\theta x, y) \in T \}, \text{ and} \quad (2)$$

$$\lambda(x, y) = \sup \{ \lambda \mid (x, \lambda y) \in T \}. \quad (3)$$

The variables $\theta \in [0,1]$ and $\lambda \in [1,+\infty)$ are the input- and output-oriented efficiency scores, respectively. These scores indicate the degree of the proportional contraction (augmentation) of inputs (outputs). The efficient points feature efficiency scores equal to unity. The latter measures render the efficient observations $(x^\theta(y), y) \in T$, where $x^\theta(y) = \theta(x, y)x$, for the input direction and $(x, y^\theta(x)) \in T$, where $y^\theta(x) = \lambda(x, y)y$, for the output direction.

In empirical studies, the set T and hence the efficiency scores are unknown (Daraio, 2005). Indeed, the quantities of interest are estimated from a random sample of the DMUs, $\mathcal{X}_K = \{(x_k, y_k) \mid k = 1, 2, \dots, K\}$. The non-parametric methods (Farrell, 1957; Charnes, 1978; Deprins, 1984) have been widely employed for efficiency analysis for they are devoid of the over-restrictive hypotheses on the DGP.

In this spirit, a certain DMU, (x_k, y_k) , defines an associated production possibility set, $\tau(x_k, y_k)$, which, under the free disposability of inputs and outputs, can be given as:

$$\tau(x_k, y_k) = \{(x, y) \in \mathfrak{R}_+^{p+q} \mid x_k \leq x, y_k \geq y\}. \quad (4)$$

The union of the individual production possibility sets (Eq. 4) results in the Free Disposal Hull (FDH) estimator of the underlying technology set, T :

$$\begin{aligned} \hat{T}_{FDH} &= \bigcup_{k=1}^K \tau(x_k, y_k) \\ &= \{(x, y) \in \mathfrak{R}_+^{p+q} \mid x_k \leq x, y_k \geq y, k = 1, 2, \dots, K\}. \end{aligned} \quad (5)$$

The efficiency scores can then be obtained by plugging Eq. 5 into Eqs. 2–3.

C. Cazals et al. (2002) and later on C. Daraio & L. Simar (2005) introduced the probabilistic description of the production process. The latter approach is of particular usefulness for estimation of the robust frontiers. The production process, thus, can be described in terms of the joint probability measure, (X, Y) on $\mathfrak{R}_+^p \times \mathfrak{R}_+^q$. This joint probability measure is completely characterized by the knowledge of the probability function $H_{XY}(\cdot, \cdot)$ defined as:

$$H_{XY}(x, y) = \Pr(X \leq x, Y \geq y). \quad (6)$$

The support of $H_{XY}(\cdot, \cdot)$ is T and $H_{XY}(x, y)$ can be interpreted as the probability for a DMU operating at (x, y) to be dominated. Note that this function is a non-standard one, with a cumulative distribution form for X and a survival form for Y .

In the input orientation, it is useful to decompose the joint probability as follows:

$$\begin{aligned} H_{XY}(x, y) &= \Pr(X \leq x \mid Y \geq y) \Pr(Y \geq y) \\ &= F_{X|Y}(x \mid y) S_Y(y), \end{aligned} \quad (7)$$

where the conditional probabilities are assumed to exist, i. e. $S_Y(y) > 0$.

The input-oriented efficiency score, $\theta(x, y)$, for $(x, y) \in T$ is defined for $\forall y | S_Y(y) > 0$ as:

$$\theta(x, y) = \inf \{ \theta | F_{XY}(\theta x | y) > 0 \} = \inf \{ \theta | H_{XY}(\theta x, y) > 0 \}. \quad (8)$$

In the latter setting, the conditional distribution $F_{XY}(\cdot | y)$ acts as the feasible set of input values, X , for a DMU exhibiting the output level y . Given the free disposability assumption, the lower boundary of this set (in a radial sense) renders the Farrell-efficient frontier.

A non-parametric estimator is obtained by replacing $F_{XY}(x | y)$ by its empirical version:

$$\hat{F}_{XY,K}(x | y) = \frac{\sum_{k=1}^K I(X_k \leq x, Y_k \geq y)}{\sum_{k=1}^K I(Y_k \geq y)}, \quad (9)$$

where $I(\cdot)$ is the indicator function.

It is due to C. Cazals et al. (2002) that the estimator given by Eq. 8 coincides with the FDH one: $\hat{\theta}_{FDH}(x, y) = \inf \{ \theta | (\theta x, y) \in \hat{T}_{FDH} \} = \inf \{ \theta | \hat{F}_{XY,K}(\theta x | y) > 0 \}$. The latter estimators, however, are the deterministic ones and therefore assume that all the observations constitute the underlying technology set, namely $\Pr((x_k, y_k) \in T) = 1$. Therefore, these estimators are sensitive to the outliers as well as the atypical observations, which can affect the lower boundary of $\hat{F}_{XY,K}(x | y)$. As a remedy to the outlier problem, C. Cazals et al. (2002) suggested considering the expected value of m variables $\{X_l\}_{l=1,2,\dots,m}$ randomly drawn from the conditional distribution $\hat{F}_{XY,K}(x | y)$ (hence the term *order- m frontier*) rather than the lower boundary of $\hat{F}_{XY,K}(x | y)$ as the benchmark. Specifically, the input order- m frontier is estimated via the following procedure (Daraio, 2007b): For a given level of output, y , we consider m i.i.d. random variables, $\{X_l\}_{l=1,2,\dots,m}$, generated by the conditional p -variate distribution function, $F_{XY}(x | y)$, and obtain the random production possibility set of order m for DMUs producing more than y :

$$\tilde{T}_m(y) = \{ (x, y') \in \mathfrak{R}_+^{p+q} | X_l \leq x, y' \geq y, l = 1, 2, \dots, m \}. \quad (10)$$

Then, the order- m input efficiency score is obtained as:

$$\theta_m(x, y) = E_{XY}(\tilde{\theta}_m(x, y) | Y \geq y), \quad (11)$$

with $\tilde{\theta}_m(x, y) = \inf \{ \theta | (\theta x, y) \in \tilde{T}_m(y) \}$ and E_{XY} being the expectation relative to the distribution $F_{XY}(\cdot | y)$. Given the order- m frontier might not include the observation under consideration (i. e. $(x, y) \notin T$), the input Farrell efficiency scores are no longer bounded to the interval $[0, 1]$ and can exceed the unity. As $m \rightarrow \infty$, however, $T_m \rightarrow T$ with $\theta_m(x, y) \rightarrow \theta(x, y)$, though only the asymptotic convergence is maintained.

The empirical estimator of $\theta_m(x, y)$ is obtained by plugging in the empirical version of $F_{XY}(\cdot | y)$:

$$\begin{aligned}
\hat{\theta}_{m,n}(x, y) &= \hat{E}_{x|Y}(\tilde{\theta}_m(x, y) | Y \geq y) \\
&= \int_0^\infty (1 - \hat{F}_{x|Y}(ux | y))^m du \\
&= \hat{\theta}_{FDH}(x, y) + \int_{\hat{\theta}_{FDH}(x, y)}^\infty (1 - \hat{F}_{x|Y}(ux | y))^m du
\end{aligned} \tag{12}$$

Instead of computing the univariate integral in Eq. 12, one can employ the Monte Carlo procedure:

1) For a given output level, y , draw a sample of size m with replacement among $x_k | y_k \geq y$ and denote this sample as $\{X_{l,b}\}_{l=1,2,\dots,m}$;

2) Compute the input-oriented efficiency scores as

$$\tilde{\theta}_m^b(x, y) = \min_{l=1,2,\dots,m} \left\{ \max_{i=1,2,\dots,p} \left\{ \frac{X_{l,b}^i}{x^i} \right\} \right\};$$

3) Redo this for $b = 1, 2, \dots, B$, where B is large;

4) Compute the estimate of the efficiency score: $\hat{\theta}_{m,n}(x, y) \approx \frac{1}{B} \sum_{b=1}^B \tilde{\theta}_m^b(x, y)$.

The standard FDH solution is based on the deterministic empirical frontier, which envelopes all the observed data points:

$$\hat{\theta}_{FDH}(x, y) = \min_{k=1,2,\dots,K|Y_k \geq y} \left\{ \max_{i=1,2,\dots,p} \left\{ \frac{X_k^i}{x^i} \right\} \right\}. \tag{13}$$

For the output orientation, the following efficiency score is computed:

$$\lambda(x, y) = \sup\{\lambda | S_{Y|X}(\lambda y | x) > 0\} = \sup\{\lambda | H_{XY}(x, \lambda y) > 0\}, \tag{14}$$

with the following empirical estimator of $S_{Y|X}(y | x)$:

$$\hat{S}_{Y|X}(y | x) = \frac{\sum_{k=1}^K I(X_k \leq x, Y_k \geq y)}{\sum_{k=1}^K I(X_k \leq x)}. \tag{15}$$

Indeed, $\hat{\lambda}_{FDH}(x, y) = \sup\{\lambda | (x, \lambda y) \in \hat{T}_{FDH}\} = \lambda(x, y) = \sup\{\lambda | \hat{S}_{Y|X}(\lambda y | x) > 0\}$.

The output order- m frontier is estimated via the following procedure (Daraio, 2007b): For a given level of input, x , we consider m i.i.d. random variables, $\{Y_l\}_{l=1,2,\dots,m}$, generated by the conditional q -variate distribution function, $F_{Y|X}(y | x) = \Pr(Y \leq y | X \leq x)$, and obtain the random production possibility set of order m for DMUs consuming less than x :

$$\tilde{T}_m(x) = \{(x', y) \in \mathfrak{R}_+^{p+q} | x' \leq x, y \leq Y_l, l = 1, 2, \dots, m\}. \tag{16}$$

Then, the order- m output efficiency score is obtained as:

$$\lambda_m(x, y) = E_{Y|X}(\tilde{\lambda}_m(x, y) | X \leq x), \tag{17}$$

with $\tilde{\lambda}_m(x, y) = \sup\{\lambda | (x, \lambda y) \in \tilde{T}_m(x)\}$ and $E_{Y|X}$ being the expectation relative to the distribution $F_{Y|X}(\cdot | x)$. The resulting output Farrell efficiency scores can fall below the unity.

The empirical estimator of $\lambda_m(x, y)$ is obtained by plugging in the empirical version of $S_{Y|X}(\cdot | x)$:

$$\begin{aligned}
\hat{\lambda}_{m,n}(x, y) &= \hat{E}_{Y|X}(\tilde{\lambda}_m(x, y) | X \leq x) \\
&= \int_0^\infty (1 - \hat{S}_{Y|X}(uy | x))^m du \\
&= \hat{\lambda}_{FDH}(x, y) + \int_{\hat{\lambda}_{FDH}(x, y)}^\infty (1 - \hat{S}_{Y|X}(uy | x))^m du
\end{aligned} \tag{18}$$

Instead of computing the empirical expectation in Eq. 18, one can employ the following Monte Carlo procedure:

- 1) For a given input level, x , draw a sample of size m with replacement among $y_k | x \geq x_k$ and denote this sample as $\{Y_{l,b}\}_{l=1,2,\dots,m}$;
- 2) Compute the output-oriented efficiency scores as $\tilde{\lambda}_m^b(x, y) = \min_{l=1,2,\dots,m} \left\{ \max_{j=1,2,\dots,q} \left\{ \frac{y^j}{Y_{l,b}^j} \right\} \right\}$;
- 3) Redo this for $b=1,2,\dots,B$, where B is large;
- 4) Compute the estimate of the efficiency score: $\hat{\lambda}_{m,n}(x, y) \approx \frac{1}{B} \sum_{b=1}^B \tilde{\lambda}_m^b(x, y)$.

The standard output-oriented FDH efficiency score is computed as follows:

$$\hat{\lambda}_{FDH}(x, y) = \min_{k=1,2,\dots,K | X_k \leq x} \left\{ \max_{j=1,2,\dots,q} \left\{ \frac{y^j}{Y_k^j} \right\} \right\}. \tag{19}$$

The present study used $B = 200$. The *FEAR* package (Wilson, 2008) was employed to implement the discussed measures.

Data used

The data for 200 farms selected from the FADN sample cover the period of 2004–2009. Thus a balanced panel of 1200 observations is employed for analysis. The technical efficiency was assessed in terms of the input and output indicators commonly employed for agricultural productivity analyses, see, for instance, a study by Š. Bojnec and L. Latruffe (2008). More specifically, the utilized agricultural area (UAA) in hectares was chosen as land input variable, annual work units (AWU) – as labour input variable, intermediate consumption in Litass, and total assets in Litass as a capital factor. The last two variables were deflated by respective real price indices provided by Eurostat. On the other hand, the three output indicators represent crop, livestock, and other outputs in Litass (Lt), respectively. The aforementioned three output indicators were deflated by respective price indices and aggregated into a single one. The aforementioned instance of aggregation was implemented in order to ensure that the randomly drawn output values are reasonable ones across farms of the different specialization. The analysed sample covers relatively large farms (mean UAA – 244 ha). As for labour force, the average was 3.6 AWU. One can note that crop farms were specific with the highest variation of the variables under analysis save AWU.

In order to identify the differences in efficiency across certain farming types, the farms were classified into the three groups in terms of their specialization. Specifically, farms with crop output larger than 2/3 of the total output were considered as specialized crop farms, whereas those specific with livestock output larger than 2/3 of the total output were classified as specialized livestock farms. The remaining farms fell into a residual category called mixed farming.

Results

Initially, the ordinary FDH was employed to measure the efficiency across the three farming types. Both the input and output-oriented FDH models (cf. Eqs. 13 and 19 respectively) yielded the same results: The livestock farms achieved the highest level of efficiency, viz. 92%. The mixed farms came next with the efficiency scores of 82–86% depending on the model's orientation. Finally, the crop farms featured the lowest efficiency of 79–80%.

In order to examine the sensitivity of the results, the order- m frontier was established for both input- and output-oriented models. A set of different values of m was constructed: $m = \{25, 50, 100, 250, 400, 500, 600, 750, 1000\}$. By altering the value of m one can compute the share of the observations lying outside the production frontier, whether input-oriented or output-oriented one.

The share of observations lying outside the order- m input frontier is plotted against the order of the frontier, m , in Fig. 1. For the small values of m , almost all of the observations were left out for irrespectively of the farming type. The shares of the observations outside the production frontier, though, steeply diminished with m increasing up to the value of 400. Note that the value of m indicates how many values of inputs are drawn to estimate the expected level of efficiency. For $m \geq 400$, only the share of the livestock farms outside the production frontier continued to decrease to a higher extent, whereas those associated with other farming types virtually remained stable. Specifically, some 35%, 60%, and 45% of the crop, livestock, and mixed farms respectively fell outside the production frontier at $m = 400$. These values are quite high and imply that some sort of statistical noise is present in the data. By further increasing m up to 1000, we observed the decrease in shares of the crop, livestock, and mixed farm observations outside the production frontier down to 28%, 47%, and 39% respectively. These figures resemble the proportions of the noise data in the whole dataset. Furthermore, the observations associated with the livestock farming can be considered as atypical ones in terms of the data set under analysis.

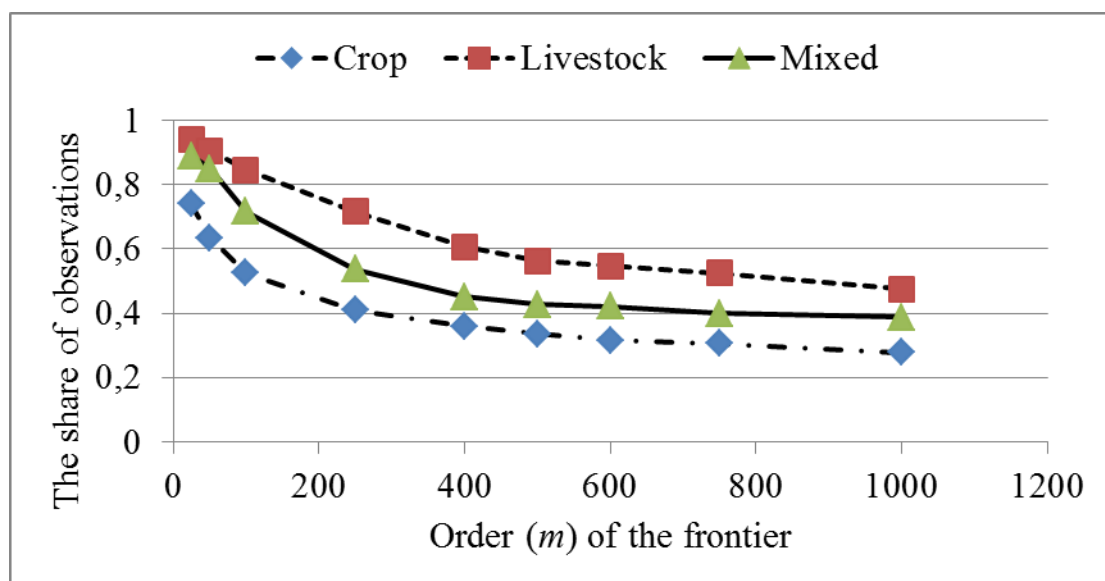


Fig. 1. The share of observations outside the input order- m frontier

As for the output order- m frontiers, they rendered much lower shares of observations outside the frontier, possibly due to the univariate output values and multivariate input vectors. The shares of the observations falling outside the frontier diminished as m increased up to 400, whereas higher values of m did not induce any significant decrease. Noteworthy, the shares of observations lying outside the production frontiers were 24%, 48%, and 10% for crop, livestock, and mixed farms respectively. At $m=1000$, these shares decreased down to 16%, 34%, and 8%. Note that in the output-oriented case the mixed farms exhibited the lowest share of observations lying outside the production frontier.

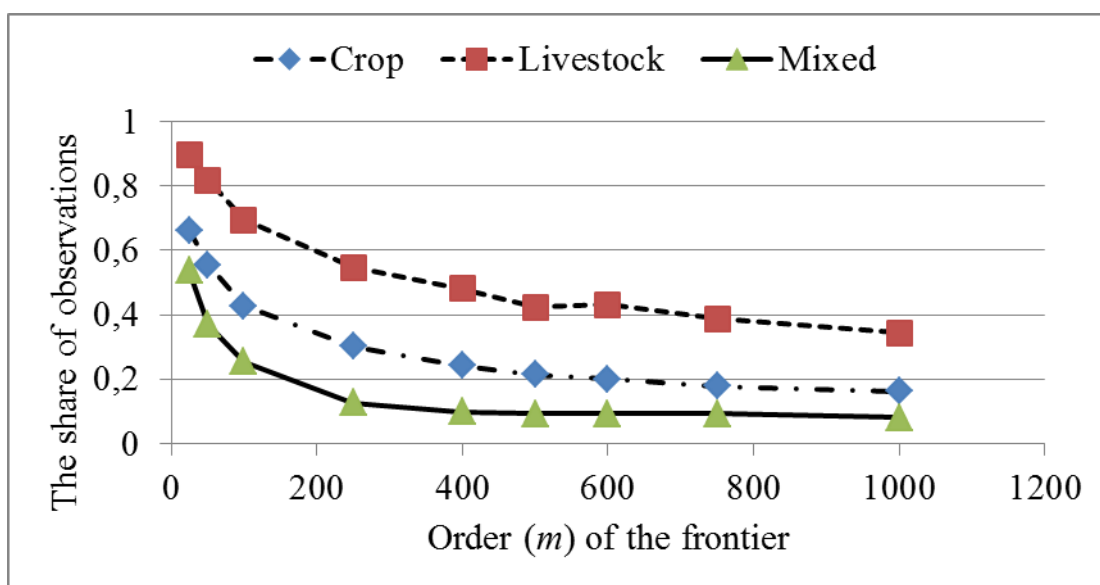


Fig. 2. The share of observations outside the output order- m frontier

Thus, one can consider the value of 400 as the order of the partial input and output production frontiers to ensure the robustness of the analysis. Indeed, frontiers with orders $m \geq 400$ exhibit similar shares of observations outside them and the only effects remaining are those of the outlier observations.

The following Figs. 3 and 4 depict the mean efficiency scores for the input- and output-oriented models. Note that the latter results are the Farrell measures (cf. Eqs. 2 and 3 for the general case; whereas Eqs. 11 and 17 correspond to the order- m estimates).

The input-oriented Farrell efficiency scores below unity indicate that a certain farm should reduce their inputs by the respective factor. On the contrary, the order- m frontiers allow for efficiency scores exceeding unity and therefore indicating that certain farms are super-efficient ones. For small m s, the mean values of the input-oriented efficiency scores exceeded unity thus indicating that most of the observations fell outside the production frontier. Anyway, the livestock farming remained the most efficient farming type at all levels of m (Fig. 3). The mixed farms exhibited slightly lower mean efficiency scores. Finally, the crop farms remained at the very bottom in terms of the mean efficiency scores. Note that the mean efficiency scores did not vary with m for the input frontier orders exceeding the value of 400.

The patterns of efficiency had somehow altered in regards to the output-oriented frontiers. Fig. 4 depicts the Farrell output efficiency scores which exceed unity in case a farm is inefficient and approaches unity as a farm gets more efficient.

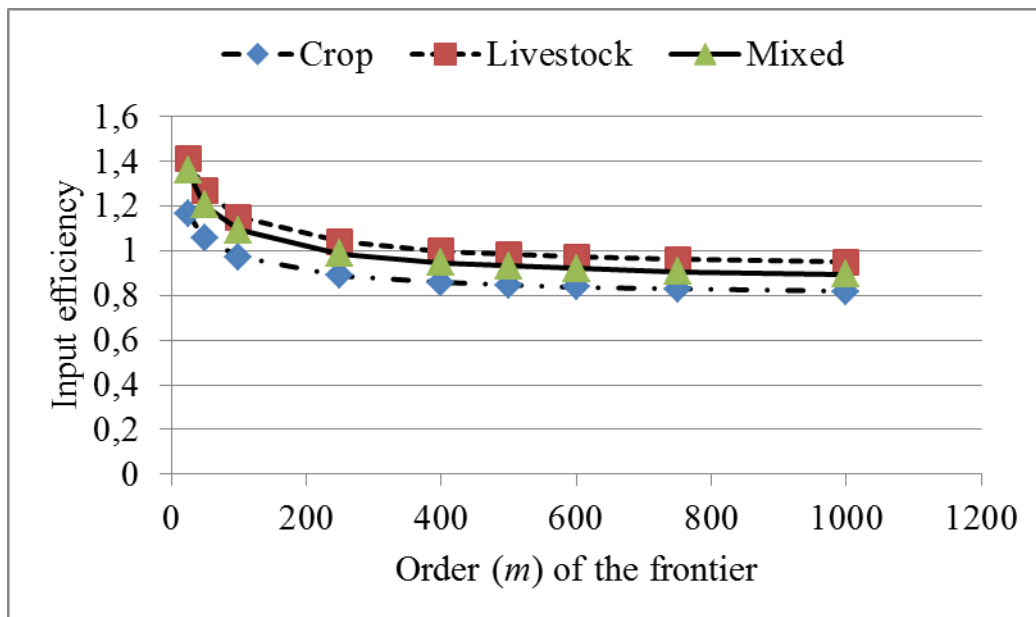


Fig. 3. The mean input Farrell efficiencies at different values of m

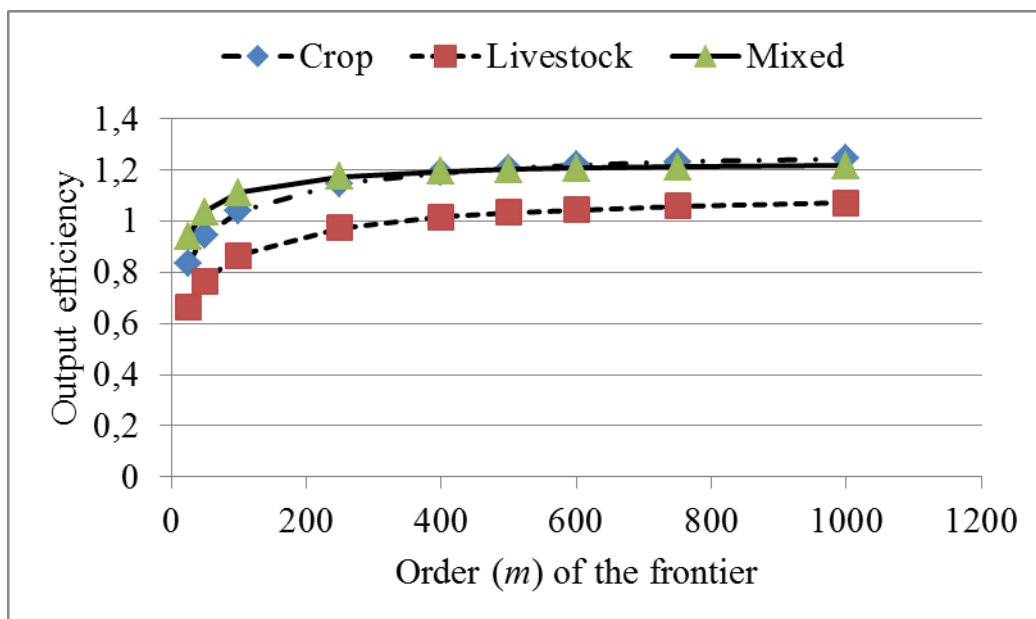


Fig. 4. The mean output Farrell efficiencies at different values of m

Those farms featuring output efficiency scores below unity are considered as super-efficient ones. This time, the crop and mixed farms exhibited extremely similar values of the mean efficiency scores: For small values of m ($m \leq 100$) the mixed farms featured the lowest efficiency scores, whereas the crop farms superseded them for $m \geq 500$. Anyway, the difference between these means remained a rather insignificant one. The livestock farms remained the most efficient ones for each value of m .

Given the discussed findings we chose the order of the production frontiers as $m = 400$ and further analysed the distributions of the efficiency scores associated with

the different farming types. Therefore, Figs. 5 and 6 present the kernel densities for the efficiency scores.

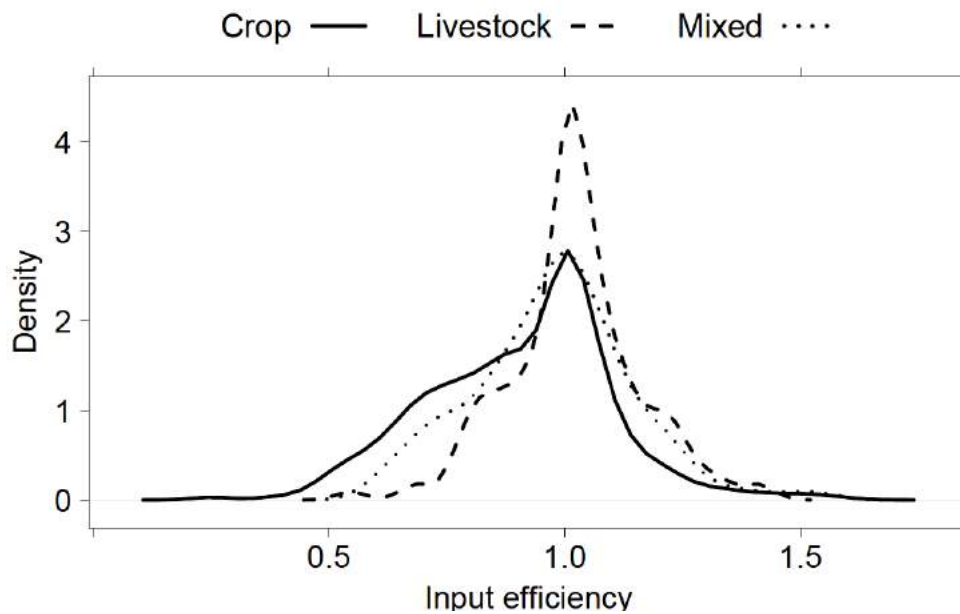


Fig. 5. The densities of the input-oriented Farrell efficiency scores ($m=400$)

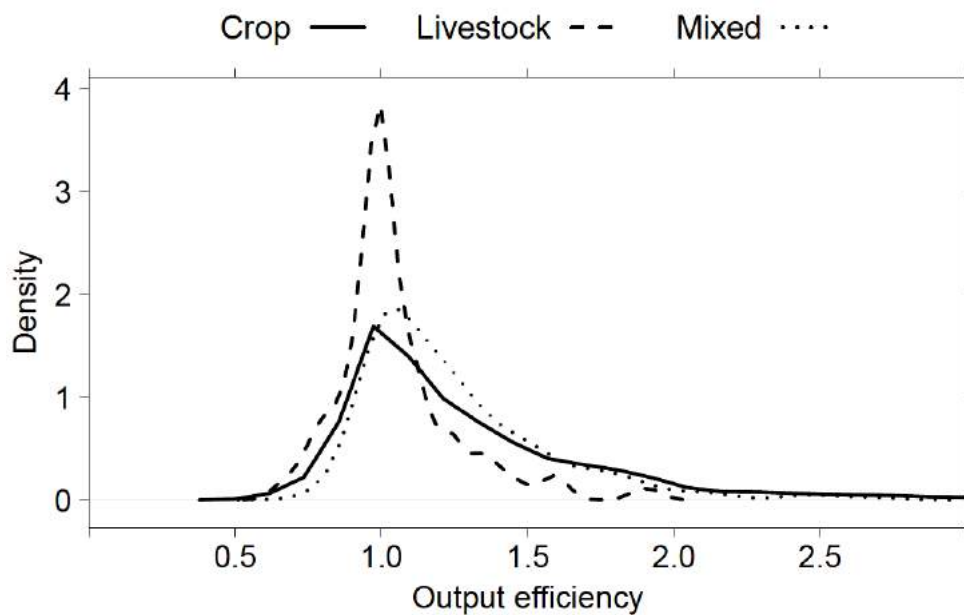


Fig. 6. The densities of the output-oriented Farrell efficiency scores ($m=400$)

As for the input efficiency scores (Fig. 5), all the farming types featured the modal values close to unity. Obviously, the livestock farms were specific with the highest concentration of the efficiency scores equal or greater to unity. Accordingly, the mean efficiency score for the livestock farms was 1.01, i. e. an average farm was super-efficient. The corresponding values for the crop and mixed farms were 0.91 and 0.98 respectively. The first quartiles for the crop, livestock, and mixed farms were 0.77, 0.95, and 0.87 respectively. Meanwhile, the third quartiles were 1.02, 1.08, and 1.54 in that order. The latter numbers can be interpreted as the minimal factor to which top 25% efficient farms could increase their consumption of inputs given

their production level and still remain efficient ones. Although the most efficient farms were the crop farms, they constituted rather insignificant share of the whole sample. Note that the maximal efficiency exceeded unity. Therefore, we can even speak of super-efficiency at this point.

The output efficiency scores were distributed in a similar way (Fig. 6). The livestock farms exhibited the most concentrated distribution. The mean values of the efficiency scores did not fall below unity for either farming type: 1.32 for the crop farms, 1.05 for the livestock farms, and 1.25 for the mixed farms. However, the first quartile for the livestock farms was 0.95 and thus indicated that more than 25% of the livestock farms were super-efficient ones. The corresponding values for the remaining farming types were ones. The third quartiles were 1.46, 1.11, and 1.38 for the crop, livestock, and mixed farms respectively.

Conclusions

1. The sensitivity of the efficiency estimates for the Lithuanian family farms was analysed by manipulating the numbers of randomly drawn benchmark observations estimations and thus constructing respective order- m frontiers. The livestock farms appeared to be most efficient, or even super-efficient, independently of the model orientation or the order of the frontier. The crop farms exhibited the lowest mean efficiency as well as the widest distribution of the efficiency scores. The latter finding might be attributed to the stochastic nature of the crop farming.

2. For instance, the shares of observations lying outside the input (resp. output) production frontier were 35%, 60%, and 45% (resp. 24%, 48%, and 10%) for crop, livestock, and mixed farms respectively at $m = 400$. These figures imply that a significant share of the livestock farm observations fell outside the production frontier and the latter farming type is a relatively efficient one. Furthermore, the mean input efficiency scores for the crop, livestock, and mixed farms were 0.91, 1.01, and 0.98, respectively. This implies that an average livestock farm was super-efficient and could have increased the consumption of inputs by 1% without any loss in efficiency. The inverse mean output efficiency scores were 0.78 for the crop farms, 0.95 for the livestock farms, and 0.8 for the mixed farms. Noteworthy, suchlike patterns of efficiency were observed across different values of order of the frontier.

3. The mixed farming was more related to the livestock farming in case of the input-oriented framework and to the crop farming in the output-oriented one in terms of the mean efficiency scores. The model's orientation, thus, is quite important option revealing certain systematic variations in the context of the analysis of the agricultural efficiency in Lithuania. In addition, these findings showed that the mixed farms were located inside the production frontier (surface) in a rather compact way.

4. The following directions can be given for the further studies: The partial frontiers of order- α , can be employed to analyse the farming efficiency. Both order- m and order- α measures should be implemented alongside the Malmquist index to measure the changes in the total factor productivity. Finally, each of the farming types could be analysed independently.

Acknowledgments

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LIETUVOS ŪKININKŲ ŪKIŲ VEIKLOS EFEKTYVUMO VERTINIMAS TAIKANT M-TOSIOS EILĖS RIBAS

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Santrauka

Straipsnyje analizuojamas Lietuvos ūkininkų ūkių veiklos efektyvumas įvertinant duomenų neapibrėžtumą. Minėtam tikslui pasiekti pritaikytos tikimybinės gamybos funkcijos. Gautųjų efektyvumo įverčių jautrumo analizė buvo atlikta keičiant atsitiktinės atskaitos ūkių imties dydį, taip suformuojant atitinkamas m-osios eilės gamybos ribas. Gyvulininkystės ūkiai veikė santykinai efektyviausiai, nepriklausomai nuo modelio orientacijos į išteklių taupymą ar produkcijos apimties didinimą, ir atsitiktinės imties dydžio. Augalininkystės ūkiai pasižymėjo žemiausiais vidutiniais efektyvumo įverčiais ir didžiausia duomenų sklaida. Svarbu paminėti, kad pačiais didžiausiais efektyvumo įverčiais pasižymėjo atskiri augalininkystės ūkiai, taigi neefektyvumas minėtame ūkininkavimo tipe gana dažnai gali būti lemiamas gamtinių ar vadybinių aplinkybių. Mišrūs ūkiai efektyvumo lygio atžvilgiu buvo panašesni į gyvulininkystės ūkius į išteklių taupymą orientuotame modelyje, o į augalininkystės ūkius – į produkcijos apimties didinimo modelyje. Taigi mišrūs ūkiai yra susikoncentravę efektyvumo ribos viduje ir projektuojami ant skirtingų ribos (paviršiaus) plokštumų keičiant modelio orientaciją.

Raktiniai žodžiai: efektyvumas, ūkininkų ūkiai, m-tosios eilės riba, veiklos analizė.

JEL kodai: C140, C440, D240, Q120.