PROBABILISTIC PRODUCTIVE TECHNOLOGY MODEL AND PARTIAL PRODUCTION FRONTIERS: AN APPLICATIONFOR LITHUANIAN AGRICULTURE

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Introduction

The measurement of productive efficiency is one of the key issues of economic studies. Indeed, the changes in efficiency might result in the release of certain factors of production as well as changes in prices. The frontier methods are the primary tool for efficiency analysis. Indeed, they can be grouped with respect to their features. First, the frontier methods can be either parametric or non-parametric. Parametric methods, e. g. stochastic frontier analysis, require certain assumptions in regards to the underlying functional form and the distribution of error terms. On the other hand, the non-parametric methods, e. g. free disposal hull (FDH) or data envelopment analysis (DEA), do not require suchlike assumptions, but rely on the implicit production (cost) functions. For instance, DEA presumes a piece-wise linear production function. Second, the frontier methods can be grouped with stochastic and deterministic ones. The stochastic frontier methods explain the distance between an observation and the production frontier as a convolution of inefficiency and random error, whereas the deterministic ones treat the same distance purely as an outcome of either inefficiency or random error. The axiomatic methods enable imposing certain axioms regarding the production frontier. For instance, DEA estimates the production frontier with respect to the axioms of concavity, monotonicity and minimal extrapolation (Afriat, 1972). Non-axiomatic methods (ordinary least squares, stochastic frontier analysis) might estimate production frontiers which do not satisfy the aforementioned axioms. Therefore, there is a need for the application of various frontier techniques as well as integration thereof in order to ensure a robust measurement of efficiency.

The following circumstances stress the need for efficiency measurement in the agricultural sector. First, the agricultural sector is related to voluminous public support. Second, a significant share of the rural population is employed in the agricultural sector. This is particularly the case in Central and Eastern European countries, where agricultural sectors are even more important for the local economies. The measures of efficiency enable us to describe the state of agricultural sector as well as identify the means for improvement (Mendes et al., 2013).

The analyses of efficiency and productivity usually rest on the estimation of the production frontier. The production frontier can be estimated via either the parametric or non-parametric methods or combinations thereof. The non-parametric techniques are appealing due to the fact that they do not need the explicit assumptions on the functional form of the underlying production function and still enable the imposition of certain axioms in regard to the latter function (Afriat, 1972).

The deterministic non-parametric methods, though, feature some caveats. Given the data generating process (DGP) of the observed production set is unknown, the underlying production set also remains unknown. Therefore, the efficiency scores based on the observed data—which constitute a single realization of the underlying DGP—might be biased due to outliers. As a remedy to the latter shortcoming, the statistical inference could be employed to construct the random production frontiers.

The partial frontiers (also referred to as the robust frontiers) were introduced by Cazals et al. (2002). The idea was to benchmark an observation not against all the observations dominating it but rather against a randomly drawn sample of these. This type of frontier was named the order–*m* frontier. The latter methodology has been extended by introducing the conditional measures enabling the analysis of the impact of the environmental variables on the efficiency scores (Daraio, Simar, 2005, 2007a, 2007b). Wheelock and Wilson (2003) introduced the Malmquist productivity index based on the partial frontiers. Simar and Vanhems (2013) presented the directional distance functions in the environment of the partial frontiers. The order–*m* frontiers have been employed in the sectors of healthcare (Pilyavsky, Staat, 2008) and finance (Abdelsalam et al., 2013), among others.

The order–α frontiers were introduced to define the benchmark by setting the probability of dominance, α. It was Aragon et al. (2005) who introduced the concept of the order–α frontiers in a (partially) univariate framework. Daouia and Simar (2007) further developed the latter concept by allowing for the multivariate analysis. Wheelock and Wilson (2008) offered an unconditional measure of the α–efficiency.

The object of the research is for Lithuanian family farms reporting to the Farm Accountancy Data Network. **The aim of the research** is to analyse the efficiency of the Lithuanian family farms by employing the partial production frontiers. **Research methods**: the partial frontier methodology was employed for the research. Specifically, order–m and order–α frontiers were applied for the analysis. The kernel estimates of the densities of the efficiency scores were also used.

Deterministic and probabilistic production technology

The activity analysis (Koopmans, 1951; Debreu, 1951) defines the production technology by treating the sets of inputs, $x \in \mathbb{R}_+^p$, and outputs, $y \in \mathbb{R}_+^q$, across the decision making units (DMUs). The technology set, *T*, consists of all feasible production plans:

$$T=\left\{(x,y)\in\mathbb{R}_{+}^{p+q}\mid x\text{ can produce }y\right\}. \qquad (1)$$

Furthermore, the free disposability of inputs and outputs is assumed (Shepard, 1970), i. e. $(x,y)\in T\Rightarrow(x',y')\in T$ for $x\le x', y'\le y$. Note that inequalities between vectors are to be read element-wise throughout the paper.

The input- and output-oriented Farrell measures of efficiency can be defined, respectively, as (Farrell, 1957):

$$\theta(x,y)=\inf\left\{\theta\mid(\theta x,y)\in T\right\}, \text{ and} \qquad (2)$$

$$\lambda(x,y)=\sup\left\{\lambda\mid(x,\lambda y)\in T\right\}. \qquad (3)$$

The variables $\theta\in[0,1]$ and $\lambda\in[1,+\infty)$ are the input– and output–oriented efficiency scores, respectively. These scores indicate the degree of the proportional contraction (augmentation) of inputs (outputs). The efficient points feature efficiency scores equal to unity. The latter measures render the efficient observations $\left(x^{\partial}(y),y\right)\in T$, where $x^{\partial}(y)=\theta(x,y)x$, for the input direction and $\left(x,y^{\partial}(x)\right)\in T$, where $y^{\partial}(x)=\lambda(x,y)y$, for the output direction.

In empirical studies, the set *T* and hence the efficiency scores are unknown (Daraio, Simar, 2005). Indeed, the quantities of interest are estimated from a random sample of the *K* DMUs, $\chi_K=\left\{(x_k,y_k)\mid k=1,2,...,K\right\}$. The non-parametric methods (Farrell, 1957; Charnes et al., 1978; Deprins et al., 1984) have been widely employed for efficiency analysis for they are devoid of the over-restrictive hypotheses on the DGP.

In this spirit, a certain DMU, (x_k,y_k), defines an associated production possibility set, $\tau(x_k,y_k)$, which, under the free disposability of inputs and outputs, can be given as:

$$\tau(x_k,y_k)=\left\{(x,y)\in\mathbb{R}_{+}^{p+q}\mid x_k\le x, y_k\ge y\right\}. \qquad (4)$$

The union of the individual production possibility sets (Eq. 4) results in the Free Disposal Hull (FDH) estimator of the underlying technology set, *T*:

$$\begin{aligned}\hat{T}_{FDH}&=\bigcup_{k=1}^{K}\tau(x_k,y_k)\\&=\left\{(x,y)\in\mathbb{R}_{+}^{p+q}\mid x_k\le x, y_k\ge y,\forall k=1,2,...,K\right\}\end{aligned} \qquad .(5)$$

The efficiency scores can then be obtained by plugging Eq. 5 into Eqs. 2-3. The resulting FDH estimators can be a min-max numerical problem, an integer programming problem, or a probabilistic numerical problem.

Note that a certain production plan, (x,y), consists of the two (multi-dimensional) vectors, i. e. $x=(x_1,x_2,...,x_p)$ and $y=(y_1,y_2,...,y_q)$. Sub-indexes *k* can be added to these notations in case we speak of some real observations. The min-max problem (Deprins et al., 1984) benchmarks a certain DMU against the dominating ones in terms of the most favourable input (output). The following estimators of θ and λ are obtained:

$$\hat{\theta}_{FDH}(x,y)=\min_{k\mid y\le y_k}\left\{\max_{i=1,2,...,p}\left\{\frac{x_{ik}}{x_i}\right\}\right\}, \text{ and} \qquad (6)$$

$$\hat{\lambda}_{FDH}(x,y)=\max_{k\mid x_k\le x}\left\{\min_{j=1,2,...,q}\left\{\frac{y_j}{y_{jk}}\right\}\right\}. \qquad (7)$$

The corresponding estimator of θ can also be given as an integer programming problem. The latter problem projects a certain observation onto the production frontier. In the framework of FDH, the production frontier is a non-convex hull (cf. Eq. 5). Thus, the following problem renders an estimate of θ:

$$\hat{\theta}_{FDH}(x,y)=\min_{\theta,\lambda_k}\theta$$

s. t.

$$\begin{aligned}&\sum_{k=1}^{K}\lambda_k x_{ik}\le\theta x_i,\quad i=1,2,...,p;\\&\sum_{k=1}^{K}\lambda_k y_{jk}\ge y_j,\quad j=1,2,...,q;\\&\sum_{k=1}^{K}\lambda_k=1;\\&\lambda_k\in\{0,1\},\quad k=1,2,...,K;\\&\theta\text{ unrestricted};\end{aligned} \qquad (8)$$

The input-oriented estimator would maximize the output level.

Cazals et al. (2002) and later on Daraio and Simar (2005) introduced the probabilistic description of the production process. The latter approach is of particular usefulness for estimation of the robust frontiers. The production process, thus, can be described in terms of the joint probability measure, (X,Y) on $\mathbb{R}_{+}^{p}\times\mathbb{R}_{+}^{q}$. This joint probability measure is completely characterized by the knowledge of the probability function $H_{XY}(\cdot,\cdot)$ defined as:

$$H_{XY}(x,y)=\Pr(X\le x, Y\ge y). \qquad (9)$$

The support of $H_{XY}(\cdot,\cdot)$ is *T* and $H_{XY}(x,y)$ can be interpreted as the probability for a DMU operating at (x,y) to be dominated. Note that this function is a non-standard one, with a cumulative distribution form for *X* and a survival form for *Y*.

In the input orientation, it is useful to decompose the joint probability as follows:

$$\begin{aligned}H_{XY}(x,y)&=\Pr(X\le x\mid Y\ge y)\Pr(Y\ge y)\\&=F_{X|Y}(x\mid y)S_Y(y)\end{aligned}, \qquad (10)$$

where the conditional probabilities are assumed to exist, i. e. $S_Y(y)>0$.

The input-oriented efficiency score, $\theta(x,y)$, for $(x,y)\in T$ is defined for $\forall y\mid S_Y(y)>0$ as:

$$\theta(x,y)=\inf\left\{\theta\mid F_{X|Y}(\theta x\mid y)>0\right\}=$$

$$=\inf\left\{\theta\mid H_{XY}(\theta x,y)>0\right\}. \qquad (11)$$

In the latter setting, the conditional distribution $F_{X|Y}(\cdot|y)$ acts as the feasible set of input values, *X*, for a DMU exhibiting the output level *y*. Given the free disposability assumption, the lower boundary of this set (in a radial sense) renders the Farrell-efficient frontier.

A non-parametric estimator is obtained by replacing $F_{X|Y}(x|y)$ by its empirical version:

$$\hat{F}_{X|Y,K}(x|y) = \frac{\sum_{k=1}^{K} I(X_k \le x, Y_k \ge y)}{\sum_{k=1}^{K} I(Y_k \ge y)}, \quad (12)$$

where $I(\cdot)$ is the indicator function.

Partial production frontiers

It is due to Cazals et al. (2002) that the FDH estimator of the input efficiency given by Eq. 2 is

$$\hat{\theta}_{FDH}(x,y) = \inf\{\theta | (\theta x, y) \in \hat{T}_{FDH}\} =$$
$$= \inf\{\theta | \hat{F}_{X|Y,K}(\theta x|y) > 0\}. \quad (13)$$

The latter estimator, however, is a deterministic one and therefore assumes that all the observations constitute the underlying technology set, namely $\Pr((x_k, y_k) \in T) = 1$. Therefore, these estimators are sensitive to the outliers as well as the atypical observations, which can affect the lower boundary of $\hat{F}_{X|Y,K}(x|y)$. As a remedy to the outlier problem, Cazals et al. (2002) suggested considering the expected value of *m* variables $\{X_l\}_{l=1,2,\dots,m}$ randomly drawn from the conditional distribution $\hat{F}_{X|Y,K}(x|y)$ (hence the term *order–m frontier*) rather than the lower boundary of $\hat{F}_{X|Y,K}(x|y)$ as the benchmark.

Specifically, the input order–m frontier is estimated via the following procedure (Daraio, Simar, 2007b): For a given level of output, y, we consider *m* i.i.d. random variables, $\{X_l\}_{l=1,2,\dots,m}$, generated by the conditional *p*–variate distribution function, $F_{X|Y}(x|y)$, and obtain the random production possibility set of order m for DMUs producing more than y:

$$\tilde{T}_m(y) = \{(x, y') \in \mathbb{R}_+^{p+q} | X_l \le x, y' \ge y, l =$$
$$1,2,\dots,m\}. \quad (14)$$

Then, the order–m input efficiency score is obtained as:

$$\theta_m(x,y) = E_{X|Y}(\tilde{\theta}_m(x,y) | Y \ge y), \quad (15)$$

with $\tilde{\theta}_m(x,y) = \inf\{\theta | (\theta x, y) \in \tilde{T}_m(y)\}$ and $E_{X|Y}$ being the expectation relative to the distribution $F_{X|Y}(\cdot|y)$. Given the order–m frontier might not include the observation under consideration (i. e. $(x,y) \notin T$), the input Farrell efficiency scores are no longer bounded to the interval $[0,1]$ and can exceed the unity. As $m \to \infty$, however, $T_m \to T$ with $\theta_m(x,y) \to \theta(x,y)$, though only the asymptotic convergence is maintained.

The empirical estimator of $\theta_m(x,y)$ is obtained by plugging in the empirical version of $F_{X|Y}(\cdot|y)$:

$$\hat{\theta}_{m,n}(x,y) = \hat{E}_{X|Y}(\tilde{\theta}_m(x,y) | Y \ge y)$$
$$= \int_0^\infty \left(1 - \hat{F}_{X|Y}(ux|y)\right)^m du \quad (16)$$
$$= \hat{\theta}_{FDH}(x,y) + \int_{\hat{\theta}_{FDH}(x,y)}^\infty \left(1 - \hat{F}_{X|Y}(ux|y)\right)^m du.$$

Instead of computing the univariate integral in Eq. 16, one can employ a Monte Carlo procedure (cf. Daraio, Simar, 2007b). The present study used $B = 200$ as the number of bootstrap replications. The *FEAR* package (Wilson, 2008) was employed to implement the discussed measures.

It is due to Aragon et al. (2005), that the definition of the efficiency score given by Eq. 11 is based on the order one quantile of the laws of X given $y \le Y$. Naturally, they proposed a concept of production function of continuous order $\alpha \in (0,1]$. Note that the concept of the order–*m* frontier (Cazals et al., 2002) is related to the discrete parameter, *m*. The parameter $(1-\alpha) \times 100\%$ thus indicates the probability that a certain observation is dominated by those producing at least the same amount of outputs (resp. using at most the same amount of inputs) even after the inputs (resp. outputs) are contracted (resp. augmented) with respect to the production frontier. Indeed, the underlying production remains unaltered, whereas the order–*m* frontiers are defined in terms of the randomly drawn samples.

Daouia and Simar (2007), therefore, introduced the order–α conditional efficiency measures for multi–input and multi–output technology. The α–quantile input efficiency score for the DMU $(x,y) \in T$ is defined as:

$$\theta_\alpha(x,y) = \inf\{\theta | F_{X|Y}(\theta x|y) > 1-\alpha\}, \quad (17)$$

where y is such that $S_Y(y) > 0$ and $\alpha \in (0,1]$. Similarly, the α–quantile output efficiency score for the DMU $(x,y) \in T$ is defined as:

$$\lambda_\alpha(x,y) = \sup\{\lambda | S_{Y|X}(\lambda y|x) > 1-\alpha\}, \quad (18)$$

where x is such that $F_X(x) > 0$ and $\alpha \in (0,1]$.

The measures described by Eqs. 17–18 can be estimated by plugging–in the empirical estimators (cf. Eq. 12). Therefore, the estimators of the input and output efficiency scores are

$$\hat{\theta}_{\alpha,n}(x,y) = \inf\{\theta | \hat{F}_{X|Y}(\theta x|y) > 1-\alpha\}, \quad (19)$$

$$\hat{\lambda}_{\alpha,n}(x,y) = \sup\{\lambda | \hat{S}_{Y|X}(\lambda y|x) > 1-\alpha\}. \quad (20)$$

These estimators, in turn, are computed as follows (Daouia, 2007). Let $M_y = \sum_{k=1}^{K} I(Y_k \ge y) > 0$ and define

$$\xi_k = \max_{i=1,2,\dots,p}\left\{\frac{X_k^i}{x^i}\right\}, k = 1,2,\dots,K. \quad (21)$$

For $l = 1,2,\dots,M_y$, denote by $\xi_{(l)}^y$ the permutation of the observations ξ_k such that $Y_k \ge y$: $\xi_{(1)}^y \le \xi_{(2)}^y \le \dots$ ( )
$\dots \le \xi_{(M_y)}^y$ Then we have

$$\hat{F}_{X|Y,K}(\theta x \mid y) = \frac{\sum_{k|Y_k \geq y} I(X_k \leq \theta x)}{M_y} =$$

$$= \frac{\sum_{k|Y_k \geq y} I(\xi_k \leq \theta)}{M_y} = \frac{\sum_{l=1}^{M_y} I(\xi_{(l)}^y \leq \theta)}{M_y} =$$

$$= \begin{cases} 0 \text{ if } \theta < \xi_{(1)}^y \\ l / M_y \text{ if } \xi_{(l)}^y \leq \theta \leq \xi_{(l+1)}^y, l = 1, ..., M_y - 1 \\ 1 \text{ if } \theta \geq \xi_{(M_y)}^y \end{cases} . \quad (22)$$

Accordingly,

$$\hat{\theta}_{\alpha,n}(x,y) = \begin{cases} \xi_{((1-\alpha)M_y)}^y \text{ if } (1-\alpha) M_y \in \mathbb{N}^* \\ \xi_{([(1-\alpha)M_y]+1)}^y \text{ otherwise} \end{cases}, \quad (23)$$

where $\mathbb{N}^*$ denotes the set of positive integers and $[(1-\alpha)M_y]$ denotes the integral part of $(1-\alpha)M_y$.

Note that $\hat{\lambda}_{\alpha,n}$ and $\hat{\theta}_{\alpha,n}$ converge to the associated FDH estimators as $\alpha \to 1$. The FEAR package (Wilson, 2008) was employed to obtain the quantile–based efficiency measures.

Results

The farm level data from Farm Accountancy Data Network (FADN) were used for the analysis (Lithuanian Institute of Agrarian Economics, 2010). The sample covered the years 2004–2009 and contained 1200 observations ($K = 1200$).

The order–*m* frontier was established for input–oriented models. A set of different values of *m* was constructed: $m = \{25, 50, 100, 250, 400, 500, 600, 750, 1000\}$. By altering the value of *m* one can compute the share of the observations lying outside the production frontier, whether input– or output–oriented.

The share of observations lying outside the order–*m* input frontier was related to the order of the frontier, *m*. For the small values of *m*, almost all of the observations were left out irrespective of the farming type. The shares of the observations outside the production frontier, though, steeply diminished with *m* increasing to the value of 400. Note that the value of *m* indicates how many values of inputs are drawn to estimate the expected level of efficiency. For $m \geq 400$, only the share of the livestock farms outside the production frontier continued to decrease to a higher extent, whereas those associated with other farming types virtually remained stable. Specifically, some 35%, 60%, and 45% of the crop, livestock, and mixed farms, respectively, fell outside the production frontier at $m = 400$. These values are quite high and imply that some sort of statistical noise is present in the data. By further increasing *m* to 1000, we observed the decrease in shares of the crop, livestock, and mixed farm observations outside the production frontier down to 28%, 47%, and 39%, respectively. These figures resemble the proportions of the noise data in the whole dataset. Furthermore, the observations associated with the livestock farming can be considered atypical in terms of the data set under analysis.

The following Table 1 reports the mean efficiency scores for the input–oriented models. Note that the latter results are the Farrell measures (cf. Eq. 2 for the general case; whereas Eq. 11 corresponds to the order–*m* estimates).

Table 1. ***The mean input Farrell efficiencies at different values of m***

m	Crop farms	Livestock farms	Mixed farms
25	1.17	1.41	1.36
50	1.06	1.27	1.20
100	0.97	1.15	1.09
250	0.89	1.04	0.99
400	0.86	1.00	0.85
500	0.85	0.98	0.93
600	0.84	0.97	0.92
750	0.83	0.96	0.91
1000	0.82	0.95	0.89

The input–oriented Farrell efficiency scores below unity indicate that a certain farm should reduce their inputs by the respective factor. On the contrary, the order–*m* frontiers allow for efficiency scores exceeding unity and therefore indicating that certain farms are super-efficient. For small *m*s, the mean values of the input–oriented efficiency scores exceeded unity thus indicating that most of the observations fell outside the production frontier. Nonetheless, the livestock farming remained the most efficient farming type at all levels of *m* (see Table 1). The mixed farms exhibited slightly lower mean efficiency scores. Finally, the crop farms remained at the very bottom in terms of the mean efficiency scores. Note that the mean efficiency scores did not vary with *m* for the input frontier orders exceeding the value of 400.

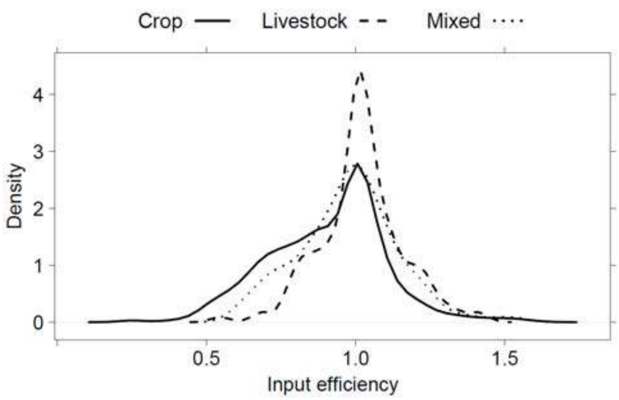

Fig. 1. The densities of the input-oriented Farrell efficiency scores (m=400)

As for the input efficiency scores (Fig. 1), all the farming types featured the modal values close to unity. Obviously, the livestock farms were specific with the highest concentration of the efficiency scores equal to or greater than unity. Accordingly, the mean efficiency score for the livestock farms was 1.01, i. e. an average farm was super–efficient. The corresponding values for the crop and mixed farms were 0.91 and 0.98 respectively. The first quartiles for the crop, livestock, and mixed farms were 0.77, 0.95, and 0.87 respectively. Meanwhile, the third quartiles were 1.02, 1.08, and 1.54 in that order. The latter numbers can be interpreted as the minimal factor to which top 25% of efficient farms could increase their consumption of inputs given their production level and still remain efficient. Although the most efficient farms were the crop farms, they constituted a rather insignificant share of the whole sample. Note that the maximal efficiency exceeded unity. Therefore, we can even speak of super–efficiency at this point.

The input–oriented measures of the order – α efficiency were implemented to analyse the farm performance with respect to different quantiles. These quantiles, indeed, enable to analysis of the variation of the observed data and estimate the level of efficiency. Let $\hat{q}_{\alpha}^{i}$ denote the input frontier (quantile) of the arbitrary order α.

The input frontiers were estimated for $\alpha = \{0.8, 0.85, 0.9, 0.95, 0.99, 0.995, 0.999, 1\}$. Note that $\hat{q}_{1}^{i}$ coincides with the full-frontier FDH estimator. Indeed, the share of farms outside the production frontier did not decrease significantly for $0 \le \alpha \le 0.95$. This finding implies that the quantiles, $\hat{q}_{\alpha}^{i}$, associated with the latter level of α were rather tight and not perturbed by the outliers. In this region the crop farms featured the highest share of the observations inside the production frontier (specifically, 17% at $\alpha = 0.95$), whereas the livestock farms were peculiar with the lowest one (2% at the same α–level). The quantiles $\hat{q}_{\alpha}^{i}$ with $\alpha \ge 0.95$ were influenced by the outliers to a higher extent and thus enveloped a higher share of the observations. At $\alpha = 0.999$, some 4%, 6%, and 16% of crop, livestock, and mixed farms remained operating outside the production frontier. Therefore, the share of the specialized crop and livestock farms diminished at a faster rate than that of mixed farms.

The mean efficiency scores were estimated for each farming types across various α–levels. Furthermore, the means were computed for input– and output–oriented models. Figs. 3 and 4 present the results for each farming type.

Table 2. ***The mean input efficiency scores for different α – levels***

α	Crop farms	Livestock farms	Mixed farms
0.8	1.91	2.39	2.51
0.85	1.74	2.16	2.24
0.9	1.55	1.94	1.95
0.95	1.33	1.65	1.60
0.99	1.03	1.22	1.16
0.995	0.94	1.11	1.05
0.999	0.81	0.94	0.90
1	0.8	0.92	0.86

The results indicated that crop farming was generally less efficient if compared to the other farming types for all values of α. An average livestock or mixed farm was super–efficient (i. e. the mean efficiency score exceeded unity) for $\alpha \le 0.995$, whereas the mean efficiency of crop farms exceeded unity at $\alpha \le 0.99$. The mixed farming was the most efficient farming type for $\alpha \le 0.9$ and the livestock farming was the most efficient farming type for $\alpha \ge 0.99$. Indeed, the difference between the mean efficiency scores further increased as the values of α approached unity. The crop farming remained the least efficient farming type in terms of the mean efficiency scores for all values of α. The FDH estimates of efficiency scores were obtained at $\alpha = 1$. The mean values were 0.8, 0.92, and 0.86 for crop, livestock, and mixed farms, respectively. These scores can be interpreted as factors of the input contraction required to ensure efficiency, e. g. an average crop farm should contract its inputs by some 20%.

Conclusions

The livestock farms appeared to be most efficient, or even super–efficient, independent of the model orientation or the order of the frontier. The crop farms exhibited the lowest mean efficiency as well as the widest distribution of the efficiency scores. The latter finding might be attributed to the stochastic nature of the crop farming.

The following directions can be given for further studies: The partial frontiers of order $-\alpha$, can be employed to analyse the farming efficiency. Both order–*m* and order–α measures should be implemented alongside the Malmquist total factor productivity index to measure the changes in the total factor productivity. Finally, each of the farming types could be analysed independently. The conditional measures are also to be employed in order to analyse the determinants of efficiency.

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PROBABILISTIC PRODUCTIVE TECHNOLOGY AND PARTIAL PRODUCTION FRONTIERS: AN APPLICATION FOR LITHUANIAN AGRICULTURE

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Summary

The paper presents the non-parametric benchmarking technique, viz. Free Disposal Hull (FDH), along with its probabilistic extensions. Indeed, the frontier methods are rather sensitive to outliers. The probabilistic methodology, therefore, is useful when handling the practical problems of benchmarking.

The paper aimed at analysing the patterns of efficiency of Lithuanian family farms with respect to the uncertain data. The latter aim was achieved by the virtue of the probabilistic production functions. The sensitivity of the efficiency scores estimated for the Lithuanian family farms was analysed by manipulating the numbers of randomly drawn benchmark observation estimations and thus constructing respective order–*m* frontiers. The livestock farms appeared to be the most efficient, or even super–efficient, independent of the model orientation or the order of the frontier. The application of the order–alpha frontier also confirmed these results.

Keywords: efficiency; family farms; partial frontier; activity analysis.

TIKIMYBINIS GAMYBOS TECHNOLOGIJOS MODELIS IR DALINĖS GAMYBOS FUNKCIJOS: TAIKYMAS LIETUVOS ŽEMĖS ŪKIO SEKTORIUJE

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Santrauka

Straipsnyje analizuojamas Lietuvos ūkininkų ūkių veiklos efektyvumas įvertinant duomenų neapibrėžtumą. Minėtam tikslui pasiekti taikytos tikimybinės gamybos funkcijos. Gautų efektyvumo įverčių jautrumo analizė atlikta keičiant atsitiktinės atskaitos ūkių imties dydį ir taip suformuojant atitinkamas m-tosios eilės gamybos ribas. Gyvulininkystės ūkiai veikė santykinai efektyviausiai nepriklausomai nuo modelio orientacijos į išteklių taupymą ar produkcijos apimties didinimą ir atsitiktinės imties dydžio. Augalininkystės ūkių vidutiniai efektyvumo įverčiai buvo mažiausi, o duomenų sklaida – didžiausia. Pažymėtina, kad pačiais didžiausiais efektyvumo įverčiais pasižymėjo atskiri augalininkystės ūkiai, taigi šio tipo ūkininkavimo neefektyvumą gana dažnai gali lemti gamtinės ar vadybinės aplinkybės. Nors efektyviausiai veikė gyvulininkystės ūkiai, pastaruoju metu Lietuvoje pastebimas gyvulių skaičiaus mažėjimas. Taigi gyvulininkystės skatinimo priemonės prisidėtų prie efektyvumo ir produktyvumo didinimo Lietuvos žemės ūkio sektoriuje.

Prasminiai žodžiai: efektyvumas, ūkininkų ūkiai, dalinė riba, veiklos analizė.

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