

MULTIMOORA-FG: A Multi-Objective Decision Making Method for Linguistic Reasoning with an Application to Personnel Selection

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Received: May 2011; accepted: September 2011

Abstract. This paper aims to extend fuzzy MULTIMOORA with linguistic reasoning and group decision-making (MULTIMOORA-FG). The new method consists of the three parts, namely the fuzzy Ratio System, the fuzzy Utopian Reference Point, and the fuzzy Full Multiplicative Form offering a robust comparison of alternatives against multiple objectives. In addition, MULTIMOORA-FG is designed to deal with triangular fuzzy numbers which, in turn, can resemble linguistic variables. MULTIMOORA-FG is a proper instrument for linguistic reasoning under fuzzy environment. In our study an application of personnel selection illustrates the group decision-making procedure according to MULTIMOORA-FG. Given the uncertainties peculiar of personnel selection, the application of multi-objective decision making (MODM) is required in this area. Fuzzy MULTIMOORA enables to aggregate subjective assessments of the decision-makers and thus offer an opportunity to perform a more robust personnel selection. The committee decided to consider eight qualitative characteristics expressed in linguistic variables. A numerical example exhibited possibilities for improvement of human resources management or any other business decision-making by applying MULTIMOORA-FG.

Keywords: personnel selection, personnel management, employment decisions, human resources management, multi-objective optimization, MULTIMOORA, MULTIMOORA-FG, fuzzy number, linguistic reasoning.

1. Introduction

Multi-objective decision-making methods (MODM) methods can deal with a selection process: by finding an optimal solution from a set of available alternatives according to a set of objectives. Sometimes none of the alternatives satisfies all the objectives. Then a *satisfactory* decision is made instead of an *optimal* one. Roy (1996) presented the following pattern of MODM problems: (1) α *choosing* problem – choosing the best alternative; (2) β *sorting* problem – classifying alternatives into relatively homogenous groups; (3) γ *ranking* problem – ranking alternatives from best to worst; (4) δ *describing* problem – describing alternatives in terms of their peculiarities and features. Belton

and Stewart (2002) defined the three broad categories of MODM methods (Loken, 2007; Zavadskas and Turskis, 2011): (1) value measurement models; (2) goal, aspiration, and reference level models; (3) outranking models (the French school). In this study we will extend and apply the MOORA and MULTIMOORA methods.

The Multi-Objective Optimization by Ratio Analysis (MOORA) was introduced by Brauers and Zavadskas (2006). Subsequently, these authors further developed the method (Brauers and Zavadskas, 2010a) thus presenting the MULTIMOORA (MOORA plus the full multiplicative form). Numerous examples of application of MULTIMOORA are present. The MULTIMOORA was applied as well as in a manufacturing and engineering environment (Kracka *et al.*, 2010; Chakraborty, 2011; Brauers *et al.*, 2008a, 2008b; Kalibatas and Turskis, 2008), as in regional development studies (Brauers and Zavadskas, 2010b, 2011b; Brauers and Ginevičius, 2009, 2010; Brauers *et al.*, 2007). The theory of dominance (Brauers and Zavadskas, 2011a) enables to summarize the ranks obtained from different parts of MULTIMOORA. Moreover, MULTIMOORA has been updated with fuzzy number theory (Brauers *et al.*, 2011).

In this study a personnel selection case will be used as an assumed example. Personnel selection is an important part of human resources management in an enterprise. It tries to find the best candidate for a well-defined vacancy also with determination of the input quality of the personnel (Dursun and Karsak, 2010). The ongoing processes of globalization as well as the increasing competition require improving the personnel selection. However, many enterprises are not ready to make available the vast amount of funds for personnel selection. Hence, it is important to develop new decision-making techniques available for enterprises asking for various technological, financial, or intellectual capacities. Consequently, more and more scientists have analyzed the practice of personnel recruitment (Zavadskas *et al.*, 2008). Indeed, the complexity of personnel selection requires the application of MODM for robust recruitment. Consequently, MODM methods were applied in many studies focused on personnel recruitment (Dursun and Karsak, 2010; Zhang, Liu, 2011; Kelemenis and Askounis, 2010; Kelemenis *et al.*, 2011; Merigo and Gil-Laufente, 2011; Zavadskas *et al.*, 2008). The latter two sources provide comprehensive reviews of studies on personnel selection.

Zadeh (1965), the Founder of fuzzy logic, proposed employing the fuzzy set theory as a modelling tool for complex systems that are hard to define exactly in crisp numbers. Fuzzy logic hence allows coping with vague, imprecise and ambiguous inputs and knowledge (Kahraman, 2008; Kahraman and Kaya, 2010). Linguistic reasoning relying on fuzzy logics was introduced by Zadeh (1975a, 1975b, 1975c) and applied in many studies (Liang, 1999; Chen, 2000; Chou *et al.*, 2008; Torlak *et al.*, 2011; Kaya and Kahraman, 2011).

This paper aims at extending the fuzzy MULTIMOORA for linguistic reasoning under group decision-making. The extended fuzzy MULTIMOORA is applied for solving personnel selection. Therefore the remaining part of the paper is organized in the following way. Section 2 describes the basics of fuzzy number theory, linguistic reasoning, and MULTIMOORA. The following Section 3 presents the extended fuzzy MULTIMOORA for group decision making. Finally, Section 4 sets forward a numerical example of personnel selection.

2. Preliminaries

2.1. The Fuzzy Set Theory and Triangular Fuzzy Numbers

Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling uncertain systems. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership, while fuzzy sets allow partial membership. The theoretical fundamentals of fuzzy set theory are overviewed by Chen (2000).

In a universe of discourse X , a fuzzy subset \tilde{A} of X is defined with a membership function $\mu_{\tilde{A}}(x)$ which maps each element $x \in X$ to a real number in the interval $[0; 1]$. The function value of $\mu_{\tilde{A}}(x)$ resembles the grade of membership of x in \tilde{A} . The higher the value of $\mu_{\tilde{A}}(x)$, the higher the degree of membership of x in \tilde{A} (Keufmann and Gupta, 1991). Noteworthy, in this study any variable with tilde will denote a fuzzy number.

A fuzzy number \tilde{A} is described as a subset of real number whose membership function $\mu_{\tilde{A}}(x)$ is a continuous mapping from the real line \Re to a closed interval $[0; 1]$, which has the following characteristics: (1) $\mu_{\tilde{A}}(x) = 0$, for all $x \in (-\infty; a] \cup [c; \infty)$; (2) $\mu_{\tilde{A}}(x)$ is strictly increasing in $[a; b]$ and strictly decreasing in $[d; c]$; (3) $\mu_{\tilde{A}}(x) = 1$, for all $x \in [b; d]$, where a, b, d , and c are real numbers, and $-\infty < a \leq b \leq d \leq c < \infty$. When $b = d$ a fuzzy number \tilde{A} is called a triangular fuzzy number (Fig. 1) represented by a triplet (a, b, c) . Triangular fuzzy numbers will therefore be used in this study to characterize the alternatives. The membership function $\mu_{\tilde{A}}(x)$ is thus defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a, \\ \frac{x-a}{b-a}, & a \leq x \leq b, \\ \frac{x-c}{b-c}, & b \leq x \leq c, \\ 0, & x > c. \end{cases} \tag{1}$$

In addition, the parameters a, b , and c in (1) can be considered as indicating respectively the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event (Torlak *et al.*, 2011).

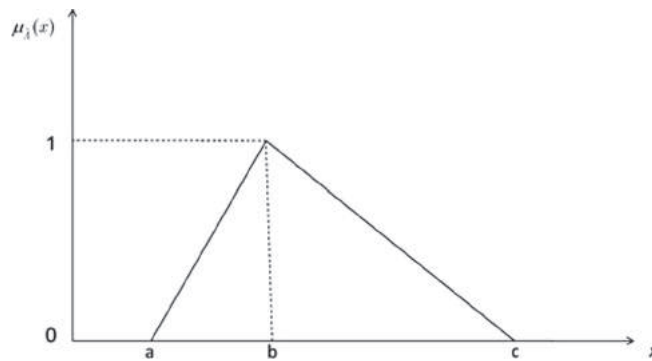


Fig. 1. Membership function of a triangular fuzzy number $\tilde{A} = (a, b, c)$.

Let \tilde{A} and \tilde{B} be two positive fuzzy numbers (Liang and Ding, 2003). Hence, the main algebraic operations of any two positive fuzzy numbers $\tilde{A} = (a, b, c)$ and $\tilde{B} = (d, e, f)$ can be defined in the following way (Merigo and Casanovas, 2011):

1. Addition \oplus :

$$\tilde{A} \oplus \tilde{B} = (a, b, c) \oplus (d, e, f) = (a + d, b + e, c + f); \quad (2)$$

2. Subtraction \ominus :

$$\tilde{A} \ominus \tilde{B} = (a, b, c) \ominus (d, e, f) = (a - f, b - e, c - d); \quad (3)$$

3. Multiplication \otimes :

$$\tilde{A} \otimes \tilde{B} = (a, b, c) \otimes (d, e, f) = (a \times d, b \times e, c \times f); \quad (4)$$

4. Division \oslash :

$$\tilde{A} \oslash \tilde{B} = (a, b, c) \oslash (d, e, f) = (a/f, b/e, c/d). \quad (5)$$

The vertex method will be applied to measure the distance between two fuzzy numbers. Let $\tilde{A} = (a, b, c)$ and $\tilde{B} = (d, e, f)$ be two triangular fuzzy numbers. Then, the vertex method can be applied to measure the distance between these two fuzzy numbers:

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3} [(a - d)^2 + (b - e)^2 + (c - f)^2]}. \quad (6)$$

Fuzzy numbers can be applied in two ways when forming the response matrix of alternatives on objectives. First, fuzzy numbers can represent the values of linguistic variables (Zadeh, 1975a, 1975b, 1975c) when deciding either on the importance of criteria or performing qualitative evaluation of alternatives. For the latter purpose Chen (2000) describes the following fuzzy numbers identifying values of linguistic variables from scale Very poor to Very good: Very poor – (0, 0, 1); Poor – (0, 1, 3); Medium poor – (1, 3, 5); Fair – (3, 5, 7); Medium good – (5, 7, 9); Good – (7, 9, 10); Very good – (9, 10, 10). Second, the fuzzy numbers can represent monetary (quantitative) terms. It can be done either through direct input of certain fuzzy numbers into the response matrix or by aggregation of raw data (e.g., time series). For example, if there are costs “approximately equal to \$ 200” estimated, the sum can be represented by triangular fuzzy number (190, 200, 210). Moreover, the fuzzy numbers can embody expected rate of growth. For example, if there is level of unemployment of 5 per cent with expected growth of 10 per cent, a triangular fuzzy number (5, 5.5, 6.1) can summarize these characteristics. As for time series data, a fuzzy number can represent the dynamics of certain indicator during past t periods:

$$\left(\min_p \{a_p\}, \left(\prod_{p=1}^t a_p \right)^{1/t}, \max_p \{a_p\} \right), \quad (7)$$

where a_p represents the value of certain indicator during period p ($p = 1, 2, \dots, t$).

The results of comparison of alternatives based on fuzzy numbers are also expressed in fuzzy numbers. The fuzzy numbers therefore need to be converted into crisp ones in order to identify the most promising alternative. There are four defuzzification methods commonly employed: (i) the centered method (or centre of area – COA); (ii) the Mean-of-maximum (MOM); (iii) the α -cut method; and (iv) the signed distance method (Zhao and Govind, 1991; Yao and Wu, 2000). In this study the COA method will be applied to obtain the Best Non-fuzzy Performance (BNP) value:

$$BNP_{\tilde{A}} = \frac{(c - a) + (b - a)}{3} + a, \quad (8)$$

where a , b and c are respectively the lower, modal, and upper values of fuzzy number $\tilde{A} = (a, b, c)$ ¹ (Triantaphyllou, 2000; Zavadskas and Antucheviciene, 2006, 2007). Moreover, the robustness as well as precision of multicriteria optimization can be improved by applying either intuitionist fuzzy numbers (Zhang and Liu, 2010) or two-tuple linguistic representation (Liu, 2009).

2.2. The MULTIMOORA Method

Multi-Objective Optimization by Ratio Analysis (MOORA) was introduced by Brauers and Zavadskas (2006) on the basis of previous research by Brauers (2004). Brauers and Zavadskas (2010a) extended the method and in this way it became more robust as MULTIMOORA (MOORA plus the full multiplicative form). These methods have been applied in numerous studies (Brauers *et al.*, 2007; Brauers and Ginevičius, 2009, 2010; Brauers and Zavadskas, 2009) focused on regional studies, international comparisons and investment management.

MOORA method begins with matrix x_{ij} where its elements x_{ij} denote i th alternative of j th objective ($i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$). MOORA method consists of two parts: the ratio system and the reference point approach. MacCrimmon (1968) defines two stages of *weighting*, namely *normalization* and voting on *significance* of objectives. The issue of weighting is discussed by Brauers and Zavadskas (2010), Zavadskas *et al.* (2010b), while the problem of normalization is analyzed by Brauers (2007) and Turskis *et al.* (2009).

The MOORA method includes internal normalization and treats originally all the objectives equally important. In principle all stakeholders interested in the issue only could give more importance to an objective. Therefore they could either multiply the dimensionless number representing the response on an objective with a significance coefficient or they could decide beforehand to split an objective into different sub-objectives (Brauers and Ginevičius, 2009).

The *Ratio System of MOORA* defines internal data normalization by comparing an alternative of an objective to all values of that objective:

¹Mode is the measurement with the maximum frequency if there is one. As there is only a lower limit and an upper limit the average of both is taken.

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, \quad (9)$$

where x_{ij}^* denotes i th alternative of j th objective after internal normalization. Usually these numbers belong to the interval $[-1; 1]$.

They are added for a maximum ($j = 1, 2, \dots, g$) or subtracted for a minimum ($j = g + 1, \dots, n$):

$$y_i^* = \sum_{j=1}^g x_{ij}^* - \sum_{j=g+1}^n x_{ij}^*, \quad (10)$$

where y_i^* denotes the internal normalized assessment of alternative i with respect to all objectives. Finally, the y_i^* are ranked in a descending order.

The *Reference Point of MOORA* is based on the ratios found in the Ratio System (9).

The reference Point can be defined in a non-subjective way as the Maximum Objective Reference Point. This approach is called realistic and non-subjective as the co-ordinates, which are selected for the reference point, are realized in one of the candidate alternatives. The alternatives A (10; 100), B (100; 20) and C (50; 50) will result in the Maximal Objective Reference Point R_m (100; 100).

The Reference Point can also be found as an Aspiration Objective Reference or a Utopian Objective Reference Point. The co-ordinates q_i of an *Aspiration Objective Vector* are formed as: $q_j \leq r_j$; with $(r_j - q_j)$ being a subjective element.

The Aspiration Objective Vector moderates the aspirations by choosing smaller co-ordinates than in the maximal objective vector. Indeed stakeholders may be more moderate in their expectations.

On the contrary, the Utopian Objective Vector gives higher values to the co-ordinates of the reference point than the maximal objective vector. Even more, a Utopian Objective Reference Point can be considered at a very large distance, rather as an ideal point not to be reached in reality. A desirable reference point as in the Utopian Objective method is then understandable for *Performance Management*, such as for student evaluation or for any performance in the private or public sector (Brauers, 2004). Alternatives striving only for a maximum and not for a minimum are then logical consequences of this application of the Utopian Objective Reference Point².

The Maximal Objective Vector is self-evident, if the alternatives are well defined, as for projects in Project Analysis and Project Planning. Some call this realism only a *Satisficing Result* or *Bounded Rationality* as it seems that the stakeholders are completely satisfied if this realistic reference point is reached (Wierzbicki, 1982; Ahituv and Spector, 1990). Therefore, the Utopian Objective Vector, more than the Maximal Objective or the Aspiration Objective Vector, offers a better response to the idea of *Multi-Objective Optimization*.

²Nevertheless a filter can be placed beforehand, e.g., only non-smokers are allowed.

The Min–Max Metric of Tchebycheff takes care of the final ranking in an ascending order:

$$\min \left(\max_j |r_j - x_{ij}^*| \right). \tag{11}$$

The Full Multiplicative Form and MULTIMOORA

Brauers and Zavadskas (2010a) proposed MOORA to be updated by the Full Multiplicative Form. The choice of a Utopian Objective Reference Point for Performance Management embodies only maximization of the full multiplicative utility function.

Overall utility of the *i*th alternative can be expressed as a dimensionless number:

$$U_i = \prod_{j=1}^n x_{ij}, \tag{12}$$

where U_i denotes the product of objectives of the *i*th alternative to be maximized with $i = 1, 2, \dots, m$.

MULTIMOORA summarizes MOORA (i.e., Ratio System and Reference Point) and the Full Multiplicative Form. Ameliorated Nominal Group and Delphi techniques can be used to reduce remaining subjectivity (Brauers and Zavadskas, 2010a).

3. The Fuzzy MULTIMOORA Method for Group Decision Making

The fuzzy MULTIMOORA was introduced by Brauers *et al.* (2011). However, the fuzzy method is further modified in this study. The fuzzy MULTIMOORA for group decision making (MULTIMOORA-FG) begins with decision matrices $\tilde{X}^k = \tilde{x}_{ij}^k = (x_{ij1}^k, x_{ij2}^k, x_{ij3}^k)$, where \tilde{x}_{ij}^k denotes *i*th alternative of the *j*th objective evaluated by the *k*th decision maker ($i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$; and $k = 1, 2, \dots, K$). Noteworthy, these variables can represent both quantitative and qualitative assessments of alternatives. Then the responses of the decision makers are aggregated by employing fuzzy weighted averaging (FWA) operator³ (Xu and Da, 2003):

$$\tilde{x}_{ij} = \sum_{k=1}^K \tilde{w}_k \tilde{x}_{ij}^k / \sum_{k=1}^K \tilde{w}_k, \tag{13}$$

where \tilde{w}_k is the fuzzy coefficient of significance for the *k*th decision maker. The equal coefficients of importance may be applied when the executive committee is homogenous, namely $\tilde{w}_k = (1/K, 1/K, 1/K)$. Hence we arrive at the fuzzy response matrix \tilde{X} with $\tilde{x}_{ij} = (x_{ij1}, x_{ij2}, x_{ij3})$ being the aggregated responses of alternatives on objectives.

³More sophisticated aggregation operators might be applied, for instance the OWA operator (Yager, 1988) and its extensions (Merigo and Casanovas, 2011; Merigo and Wei, 2011; Merigo and Gil-Laudente, 2009; Wang and Parkan, 2007; Chen and Chen, 2003).

3.1. The Fuzzy Ratio System

The Ratio System defines internal normalization of the fuzzy numbers \tilde{x}_{ij} resulting in matrix of dimensionless numbers. It is performed by comparing appropriate values of fuzzy numbers (Liu and Liu, 2010):

$$\begin{aligned} \tilde{x}_{ij}^* &= (x_{ij1}^*, x_{ij2}^*, x_{ij3}^*) \\ &= \begin{cases} x_{ij1}^* = x_{ij1} / \sqrt{\sum_{i=1}^m [(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2]}, \\ x_{ij2}^* = x_{ij2} / \sqrt{\sum_{i=1}^m [(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2]}, \\ x_{ij3}^* = x_{ij3} / \sqrt{\sum_{i=1}^m [(x_{ij1})^2 + (x_{ij2})^2 + (x_{ij3})^2]}. \end{cases} \quad \forall i, j. \end{aligned} \quad (14)$$

Computation of summarizing ratios \tilde{y}_i^* for each i th alternative comes next. For Performance Management the \tilde{x}_{ij} are only added according to (2):

$$\tilde{y}_i^* = \sum_{j=1}^g \tilde{x}_{ij}^*. \quad (15)$$

Then each ratio $\tilde{y}_i^* = (y_{i1}^*, y_{i2}^*, y_{i3}^*)$ is de-fuzzified by applying (8):

$$BNP_i = \frac{(y_{i3}^* - y_{i1}^*) + (y_{i2}^* - y_{i1}^*)}{3} + y_{i1}^*, \quad (16)$$

where BNP_i denotes the best non-fuzzy performance value of the i th alternative. Consequently, the alternatives with higher BNP values are attributed with higher ranks.

3.2. The Fuzzy Reference Point

The fuzzy Reference Point approach is based on the fuzzy Ratios obtained by (14).

The Utopian Objective Reference point \check{r} is defined, in the application under consideration, as (1, 1, 1). Performance Management does not allow that objectives should be minimized.

Every element of the ratio matrix is recalculated and final rank is given according to deviation from the reference point (6) and the Min–Max Metric of Tchebycheff:

$$\min_i \left(\max_j d(\check{r}_j, \tilde{x}_{ij}^*) \right). \quad (17)$$

Finally the candidates are ranked in an ascending order.

3.3. The Fuzzy Full Multiplicative Form

Overall utility of the i th alternative can be expressed as dimensionless number by employing (5):

$$\tilde{U}'_i = \tilde{A}_i = (A_{i1}, A_{i2}, A_{i3}) = \prod_{j=1}^g \tilde{x}_{ij}. \tag{18}$$

Formula (4) is applied when computing these variables. Since overall utility \tilde{U}'_i is a fuzzy number, (8) has to be used to rank the alternatives namely with the higher the BNP, the higher the rank of a certain alternative.

MULTIMOORA-FG summarizes fuzzy MOORA (i.e., fuzzy Ratio System and fuzzy Reference Point) and the fuzzy Full Multiplicative Form as described by the dominance theory (Brauers and Zavadskas, 2011a). The MULTIMOORA-FG is quite an effective tool for assessing sustainability of various phenomena resulting in unbiased ranking of alternatives.

4. Personnel Selection: An Assumed Example

Group decision-making according to MULTIMOORA-FG is applied for a personnel selection problem. The enterprise has formed an executive committee consisting of four decision makers (DM_1, DM_2, DM_3, DM_4). The committee has to choose the best candidate for a vacancy from four participants (A_1, A_2, A_3, A_4). The committee has decided to consider eight objectives to be fulfilled as much as possible: (1) creativity, innovation (C_1); (2) leadership (C_2); (3) strategic planning (C_3); (4) communication skills (C_4); (5) team management (C_5); (6) emotional steadiness (C_6); (7) educational background (C_7) and (8) professional experience (C_8). More specifically, these objectives are expressed in linguistic variables belonging to a seven-point scale (Table 1). A scale from 1 to seven is considered as a maximum for the human brain (Miller, 1965).

Table 1
Linguistic term set for qualitative evaluation

Linguistic term	Fuzzy number
Very low (VL)	(0, 0, 0.16)
Low (L)	(0, 0.16, 0.34)
Medium low (ML)	(0.16, 0.34, 0.5)
Moderate (M)	(0.34, 0.5, 0.66)
Medium high (MH)	(0.5, 0.66, 0.84)
High (H)	(0.66, 0.84, 1)
Very high (VH)	(0.84, 1, 1)
Ideal (UORP)	(1, 1, 1)

Table 2
Decision matrix of the four experts

		C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
DM ₁	A ₁	H	ML	VH	H	MH	VH	VH	H
	A ₂	M	H	M	MH	H	M	VH	VH
	A ₃	M	VH	L	L	VH	MH	H	VH
	A ₄	VH	L	H	ML	VH	ML	ML	M
DM ₂	A ₁	VH	L	H	VH	H	H	H	MH
	A ₂	M	VH	H	VH	VH	H	MH	MH
	A ₃	H	H	MH	M	H	MH	M	H
	A ₄	M	ML	MH	H	MH	ML	M	M
DM ₃	A ₁	M	MH	H	MH	MH	H	H	MH
	A ₂	M	H	VH	H	M	MH	VH	H
	A ₃	H	H	MH	MH	H	MH	M	ML
	A ₄	H	H	ML	M	H	L	M	M
DM ₄	A ₁	H	MH	VH	M	M	MH	MH	MH
	A ₂	H	H	MH	VH	VH	H	H	MH
	A ₃	VH	VH	H	VH	MH	H	M	VH
	A ₄	VH	VH	M	ML	H	H	ML	ML

As said above the Utopian Objective Reference Point is preferred for Performance Management ahead of the other definitions of a reference point. Therefore a Utopian Objective Reference Point (1, 1, 1) will be added as UORP to strive for. Table 1 brings the fuzzy numbers translations of the linguistic variables.

Each of the decision makers evaluated every candidate according to the eight attributes (Table 2).

Table A1 in Annex A presents the results of translation of these ratings into fuzzy numbers as described in Table 1. The FWA operator (13) with uniform coefficients of significance, namely $\tilde{w}_k = (1/4, 1/4, 1/4)$ for all k , was applied. Nevertheless it is always possible that a decision maker, for instance the head of the personnel department, is more important for this application than the other ones. At that moment his significance coefficient is for instance doubled.

Table A2 of Annex A calculates the ratios by aggregation and by applying (14).

1. *The Fuzzy Ratio System.* The \tilde{x}_{ij} are added to come to sums of ratios \tilde{y}_i^* for each i th alternative (15). Each ratio $\tilde{y}_i^* = (y_{i1}^*, y_{i2}^*, y_{i3}^*)$ is de-fuzzified by (16).

The candidates were ranked according to the fuzzy Ratio System in a descending order (Annex A Table A5). The second candidate (A₂) was considered the best one and the fourth candidate (A₄) the worst one.

2. *Reference Point Method.* A reference point was defined as the Utopian Objective Reference Point (1, 1, 1). We proceeded with calculations of deviations of each alternative from this reference point by applying (17). The candidates were ranked

Table 3
Ranking of the candidates according to MULTIMOORA-FG

	The fuzzy Ratio System	The fuzzy Reference Point	The fuzzy Full Multiplicative Form	MULTIMOORA-FG
A ₁	2	3	2	2
A ₂	1	2	1	1
A ₃	3	1	3	3
A ₄	4	4	4	4

according to the fuzzy Reference Point in an ascending order (Annex A Table A4). In this case, the third candidate was the most preferred.

3. *Full Multiplicative Form*. The Full Multiplicative Form (18) suggested the second candidate being the best one (Annex A Table A5).
4. *MULTIMOORA-FG*. The theory of dominance (Brauers and Zavadskas 2011a) was applied when summarizing the ranks provided by different parts of MULTIMOORA-FG.

The three methods of MULTIMOORA were assumed to have the same importance. It was already noticed that one decision maker could be more important than the other ones but this is not the case with the three methods of MULTIMOORA. These three methods represent all existing methods with dimensionless measures in multi-objective optimization and consequently all the three have the same significance of importance.

As shown in Table 3, the second candidate A₂ should be employed before candidate A₁ who dominates candidate A₃, on its turn dominating candidate A₄.

The numerical example exhibited possibilities for improvement of human resources management by applying MULTIMOORA-FG. However, further studies, empirical and not simulated, might be useful when based on real fieldwork.

5. Conclusion

This study extended the MULTIMOORA method itself composed of the Ratio System, the Reference Point and the Full Multiplicative Form methods to a MULTIMOORA-FG method. This proposed MULTIMOORA-FG method is suitable for tackling uncertain information, for it allows fuzzy inputs (e.g., linguistic variables identifying expert assessments), which, in turn, reduces information losses throughout computation.

Moreover, the new MULTIMOORA-FG method is extended to cope with group decision-making. A personnel selection problem illustrates the group decision-making procedure according to MULTIMOORA-FG. In our example, an executive committee consisting of four decision makers was to choose the best candidate from four participants. The committee has decided to consider eight qualitative objectives. More specifically, these objectives were expressed in linguistic variables belonging to a seven-point scale.

The theory of dominance was applied when summarizing the ranks provided by different parts of MULTIMOORA-FG, namely the fuzzy Ratio System, the fuzzy Reference Point, and the fuzzy Full Multiplicative Form. Hence, the MULTIMOORA-FG might be suitable when making business decisions which require quantitative as well as qualitative inputs. However, further studies might be useful for extending the method if it would be based on empirical data.

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Annex A. Group Decision Making According to MULTIMOORA-FG

Table A1
The ratings of the four decision-makers expressed in triangular fuzzy numbers

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈
DM ₁	A ₁	(0.66, 0.84, 1)	(0.16, 0.34, 0.5)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.84, 1, 1)	(0.66, 0.84, 1)
	A ₂	(0.34, 0.5, 0.66)	(0.66, 0.84, 1)	(0.34, 0.5, 0.66)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)	(0.34, 0.5, 0.66)	(0.84, 1, 1)
	A ₃	(0.34, 0.5, 0.66)	(0.84, 1, 1)	(, 0.16, 0.34)	(, 0.16, 0.34)	(0.84, 1, 1)	(0.5, 0.66, 0.84)	(0.84, 1, 1)
	A ₄	(0.84, 1, 1)	(, 0.16, 0.34)	(0.66, 0.84, 1)	(0.16, 0.34, 0.5)	(0.84, 1, 1)	(0.16, 0.34, 0.5)	(0.34, 0.5, 0.66)
DM ₂	A ₁	(0.84, 1, 1)	(, 0.16, 0.34)	(0.66, 0.84, 1)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)
	A ₂	(0.34, 0.5, 0.66)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.84, 1, 1)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)
	A ₃	(0.66, 0.84, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.34, 0.5, 0.66)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)
	A ₄	(0.34, 0.5, 0.66)	(0.16, 0.34, 0.5)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.16, 0.34, 0.5)	(0.34, 0.5, 0.66)
DM ₃	A ₁	(0.34, 0.5, 0.66)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)
	A ₂	(0.34, 0.5, 0.66)	(0.66, 0.84, 1)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.34, 0.5, 0.66)	(0.84, 1, 1)	(0.66, 0.84, 1)
	A ₃	(0.66, 0.84, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)	(0.34, 0.5, 0.66)	(0.16, 0.34, 0.5)
	A ₄	(0.66, 0.84, 1)	(0.66, 0.84, 1)	(0.16, 0.34, 0.5)	(0.34, 0.5, 0.66)	(0.66, 0.84, 1)	(, 0.16, 0.34)	(0.34, 0.5, 0.66)
DM ₄	A ₁	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.84, 1, 1)	(0.34, 0.5, 0.66)	(0.34, 0.5, 0.66)	(0.5, 0.66, 0.84)	(0.5, 0.66, 0.84)
	A ₂	(0.66, 0.84, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)	(0.84, 1, 1)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.5, 0.66, 0.84)
	A ₃	(0.84, 1, 1)	(0.84, 1, 1)	(0.66, 0.84, 1)	(0.84, 1, 1)	(0.5, 0.66, 0.84)	(0.66, 0.84, 1)	(0.84, 1, 1)
	A ₄	(0.84, 1, 1)	(0.84, 1, 1)	(0.34, 0.5, 0.66)	(0.16, 0.34, 0.5)	(0.66, 0.84, 1)	(0.66, 0.84, 1)	(0.16, 0.34, 0.5)

Table A4
The fuzzy Reference Point approach

	C ₁	C ₂	C ₃	C ₄	C ₅	C ₆	C ₇	C ₈	$\max_j d(\tilde{r}_j, \tilde{x}_{ij}^*)$	Rank
A ₁	0.7025	0.8205	0.6449	0.6928	0.7581	0.6604	0.6643	0.7106	0.8205	3
A ₂	0.7776	0.6608	0.7066	0.6457	0.7063	0.7073	0.6522	0.6813	0.7776	2
A ₃	0.7025	0.6491	0.7684	0.7626	0.7019	0.7073	0.7618	0.6863	0.7684	1
A ₄	0.6912	0.7759	0.7683	0.7924	0.7019	0.8285	0.8303	0.8131	0.8303	4

Table A5
The fuzzy Ratio System and the fuzzy Full Multiplicative Form

	The fuzzy Ratio System			The fuzzy Full Multiplicative Form		
	\tilde{y}_i^*	BNP_i	Rank	\tilde{U}_i'	BNP_i	Rank
A ₁	(1.86, 2.39, 2.83)	2.36009	2	(0.00001, 0.00005, 0.00022)	0.000095	2
A ₂	(1.99, 2.53, 2.91)	2.475324	1	(0.00001, 0.00009, 0.00029)	0.000132	1
A ₃	(1.78, 2.31, 2.73)	2.27359	3	(0.00001, 0.00004, 0.00018)	0.000076	3
A ₄	(1.3, 1.84, 2.3)	1.8121	4	(0, 0.00001, 0.00004)	0.000017	4

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MULTIMOORA-FG: daugiatakslio sprendimų priėmimo metodas lingvistiniam pagrindimui ir jo taikymas personalo atrankoje

Alvydas BALEŽENTIS, Tomas BALEŽENTIS, Willem K.M. BRAUERS

Šio straipsnio tikslas – pritaikyti neraiškųjų MULTIMOORA metodą lingvistiniam programavimui grupiniame sprendimų priėmime. Naujasis MULTIMOORA-FG metodas sudarytas iš trijų dalių: neraiškiosios santykių sistemos, neraiškiojo atskaitos taško, neraiškiosios pilnosios sandaugos formos. Minėtų trijų metodų taikymas leidžia atlikti efektyvų alternatyvų įvertinimą atsižvelgiant į daugelį iš anksto apibrėžtų tikslų. MULTIMOORA-FG metodas leidžia apdoroti trečiojo laipsnio neraiškiuosius skaičius, kurie gali atspindėti kiekybinę ir kokybinę informaciją, pavyzdžiui lingvistinius kintamuosius. Taigi naujasis metodas tinkamas lingvistiniam sprendimų pagrindimui. Šiame straipsnyje MULTIMOORA-FG pritaikytas apibendrinant skirtingų ekspertų nuomones ir taip atliekant atsparią nuokrypiams personalo atranką. Dėl personalo atrankos procese pasitaikančio neapibrėžtumo, dviprasmiškumo ir netikslumo, jame tikslinga taikyti daugiakriterinio vertinimo metodus bei šiuolaikines informacines technologijas. Konkreti personalo atrankos problema buvo panaudota kaip MULTIMOORA-FG metodo taikymo pavyzdys: įmonė sudarė keturių ekspertų grupę personalo atrankai, ekspertai vertino kandidatus pagal aštuonis kriterijus. Šie kokybiniai kriterijai buvo išreikšti lingvistiniais kintamaisiais. Minėtas skaitinis pavyzdys patvirtino galimybę patobulinti žmogiškųjų išteklių valdymą taikant naująjį metodą MULTIMOORA-FG.